Demo 1: Modelling a spacecraft with 2 symmetrical solar arrays ($\approx 2\ mn$ run).

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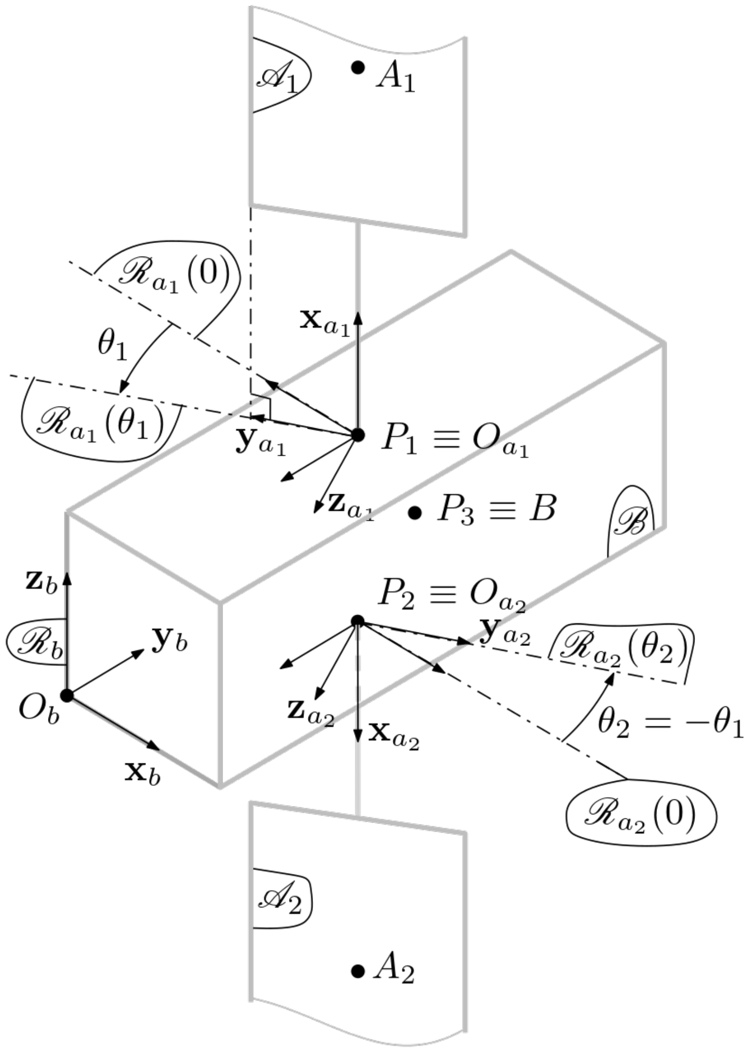
1. Description

The objective is to compute the 6×6 inverse dynamic model $[\mathbf{D}_B^{\mathcal{SC}}]^{-1}_{\mathcal{R}_b}(\mathrm{s}, \theta)$ of the spacecraft $\mathcal{SC}$ depicted in the following Figure.



The spacecraft $\mathscr{SC}$ is composed of:

- the main body $\mathscr{B}$ with its center of gravity B, its reference point O_b and its body frame $\mathscr{R}_b$,
- 2 symmetrical flexible solar arrays $\mathscr{A}_1$ and $\mathscr{A}_2$ cantilevered to $\mathscr{B}$ at the points P_1 and P_2 with an angular configuration θ_1 and θ_2, respectively. A_i, O_{a_i} and $\mathscr{R}_{a_i}$ are the centre of gravity, the reference point and the body frame of $\mathscr{A}_i$ $(i = 1, 2)$.

The angular configurations of the 2 solar panels are symmetrical: $\theta_1 = \theta$ and $\theta_2 = -\theta$. In the Figure, $\mathscr{R}_{a_i}(0)$ and $\mathscr{R}_{a_i}(\theta_i)$ represent 2 geometric configurations of $\mathscr{R}_{a_i}$ for the nominal configuration ($\theta_i = 0$) and for a given angle θ_i.

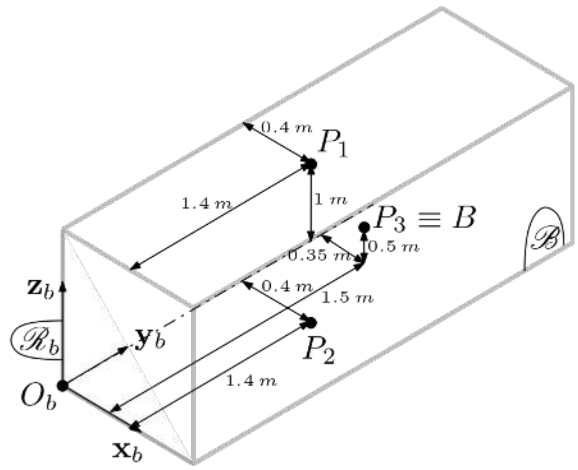
$[\mathbf{D}_B^{\mathscr{SC}}]^{-1}_{\mathscr{R}_b}(\mathrm{s}, \theta)$ is the model of the spacecraft $\mathscr{SC}$ at the centre of gravity B of the main body $\mathscr{B}$ and projected in the main body frame axes $\mathscr{R}_b$ for a given angular configuration θ, that is the 6×6 transfer between:

- the resultant external wrench $[\mathbf{W}_{ext/\mathscr{B},B}]_{\mathscr{R}_b}$ (6 components: 3 forces and 3 torques) applied to $\mathscr{B}$ at the point B,

- the dual vector of acceleration of point B $[\ddot{\mathbf{x}}_B]_{\mathscr{R}_b}$ (6 components: 3 translations, 3 rotations).

2. Required data.

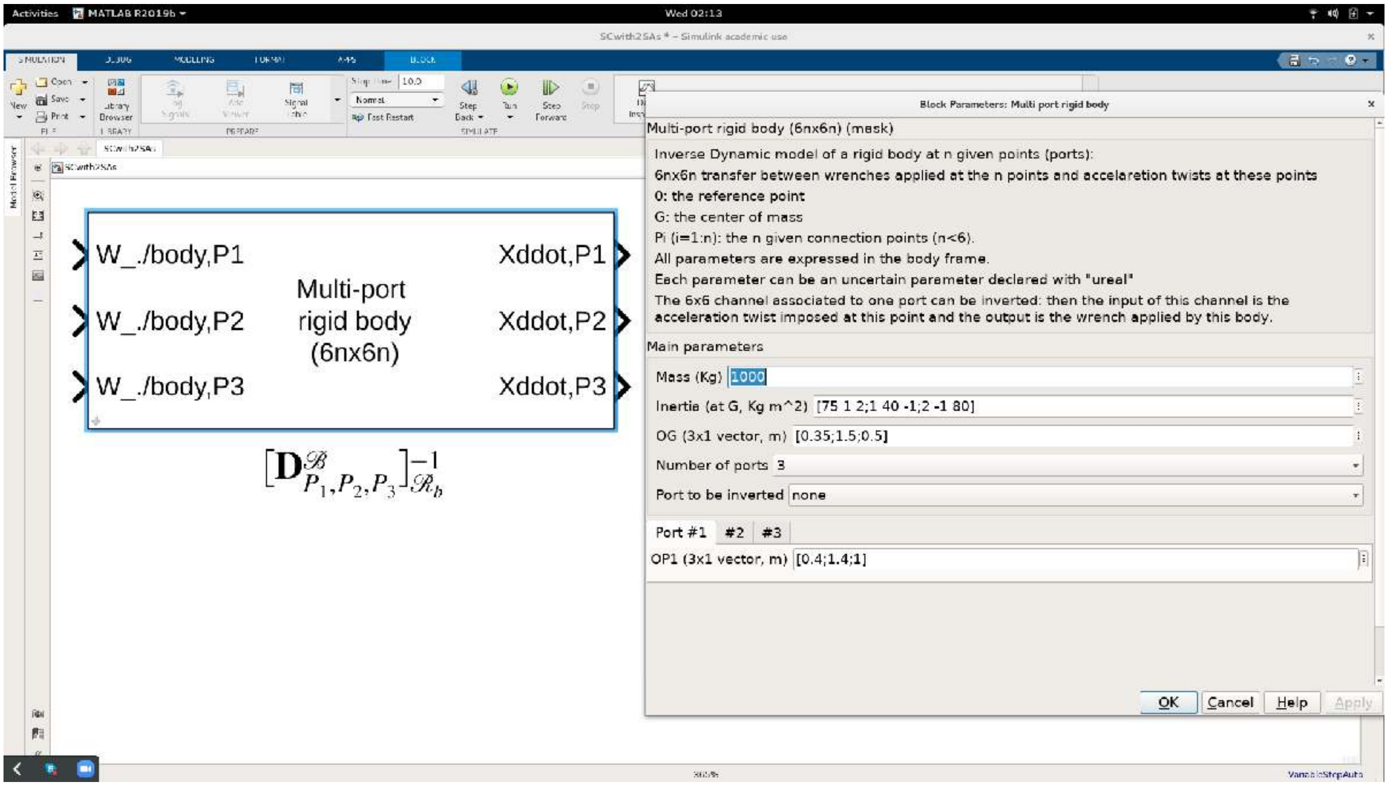
Data relative to the main body $\mathscr{B}$:



All the data are expressed in the body frame $\mathscr{R}_b$:

- geometry (in m): $[\overrightarrow{OP}_1]_{\mathscr{R}_b} = [0.4 \quad 1.4 \quad 1]^T$, $[\overrightarrow{OP}_2]_{\mathscr{R}_b} = [0.4 \quad 1.4 \quad 0]^T$, $[\overrightarrow{OB}]_{\mathscr{R}_b} = [0.35 \quad 1.5 \quad 0.5]^T$,
- mass: $m^{\mathscr{B}} = 1000\,(Kg)$,
- 3x3 inertia tensor at B: $[\mathbf{J}_B^{\mathscr{B}}]_{\mathscr{R}_b} = \begin{bmatrix} 75 & 1 & 2 \\ 1 & 40 & -1 \\ 2 & -1 & 80 \end{bmatrix}$ $(Kg\,m^2)$.

This body can be simply modeled with the block **`multi port rigid body`** of the sub-library **`6 dof bodies`** of the **`SDTlib`**. The dialog box allows to choose 3 ports (i.e. 3 points, the last port **`P3`** corresponds to the center of mass B) and to fill all the data:



The inline **help** in the dialog box opens the web browser and describes the mathematical model $[\mathbf{D}^{\mathcal{B}}_{P_1,P_2,P_3}]^{-1}$ of this block (internet connection required). This help is also avalaible by:

```
web Docmultiportrigidbody.html -new
```

Direction Cosine Matrix (DCM):

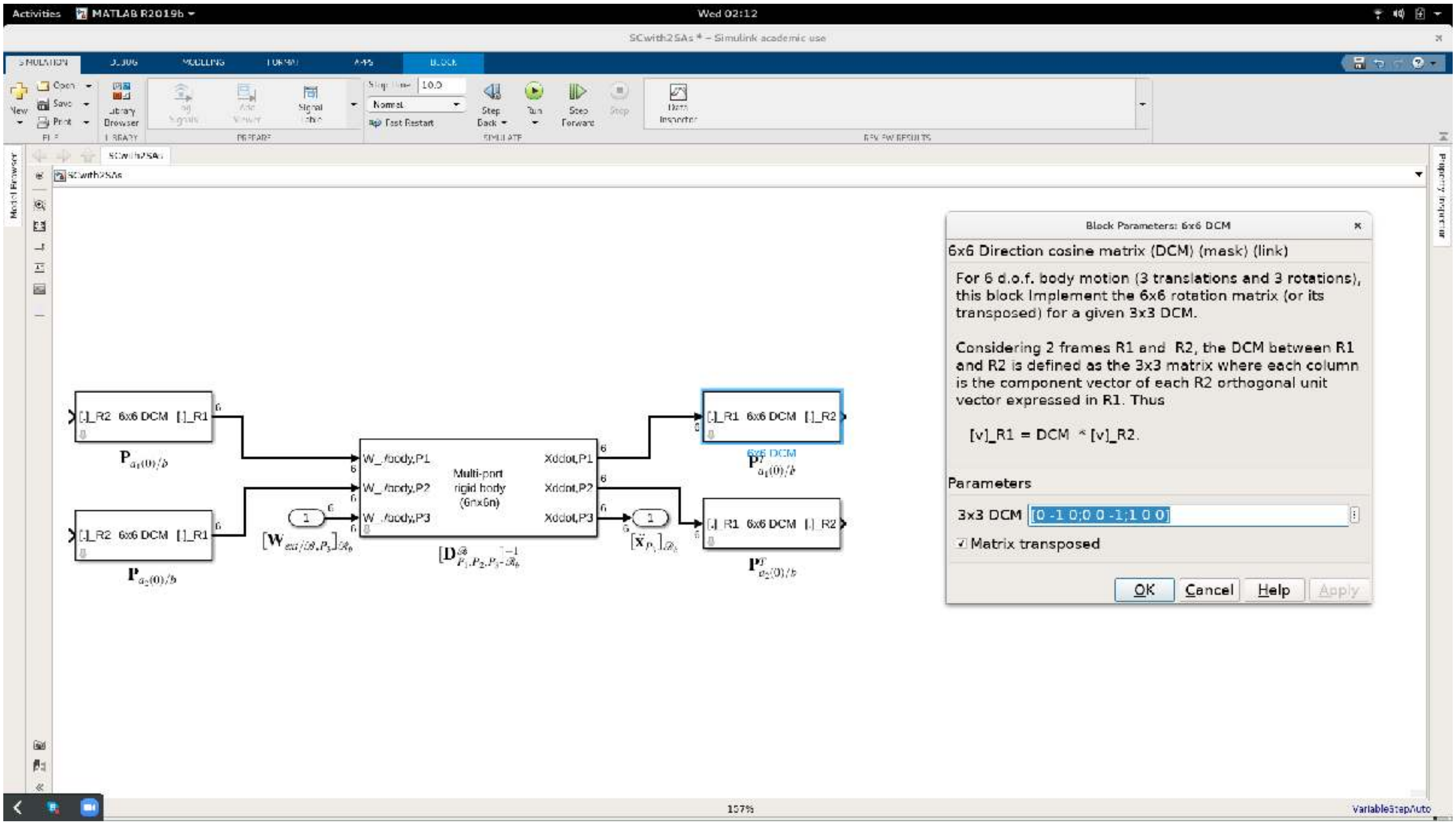
$$\mathbf{P}_{a_1(0)/b} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$ is the DCM

- from the body $\mathcal{A}_1$ frame: $\mathcal{R}_{a_1}(0)$ in the nominal configuration ($\theta = 0$)
- to the body $\mathcal{B}$ frame: $\mathcal{R}_b$.

(That is the matrix of the coordinates of $\mathbf{x}_{a_1}(0)$, $\mathbf{y}_{a_1}(0)$ and $\mathbf{z}_{a_1}(0)$ expressed in $\mathcal{R}_b$.)

In the same way: $\mathbf{P}_{a_2(0)/b} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$.

These DCMs, twiced (one for the translation, one for the rotation), can be modeled with the block **`6x6 DCM`** of the sub-library **`3/6 dof DCM`** of the **`SDTlib`**:



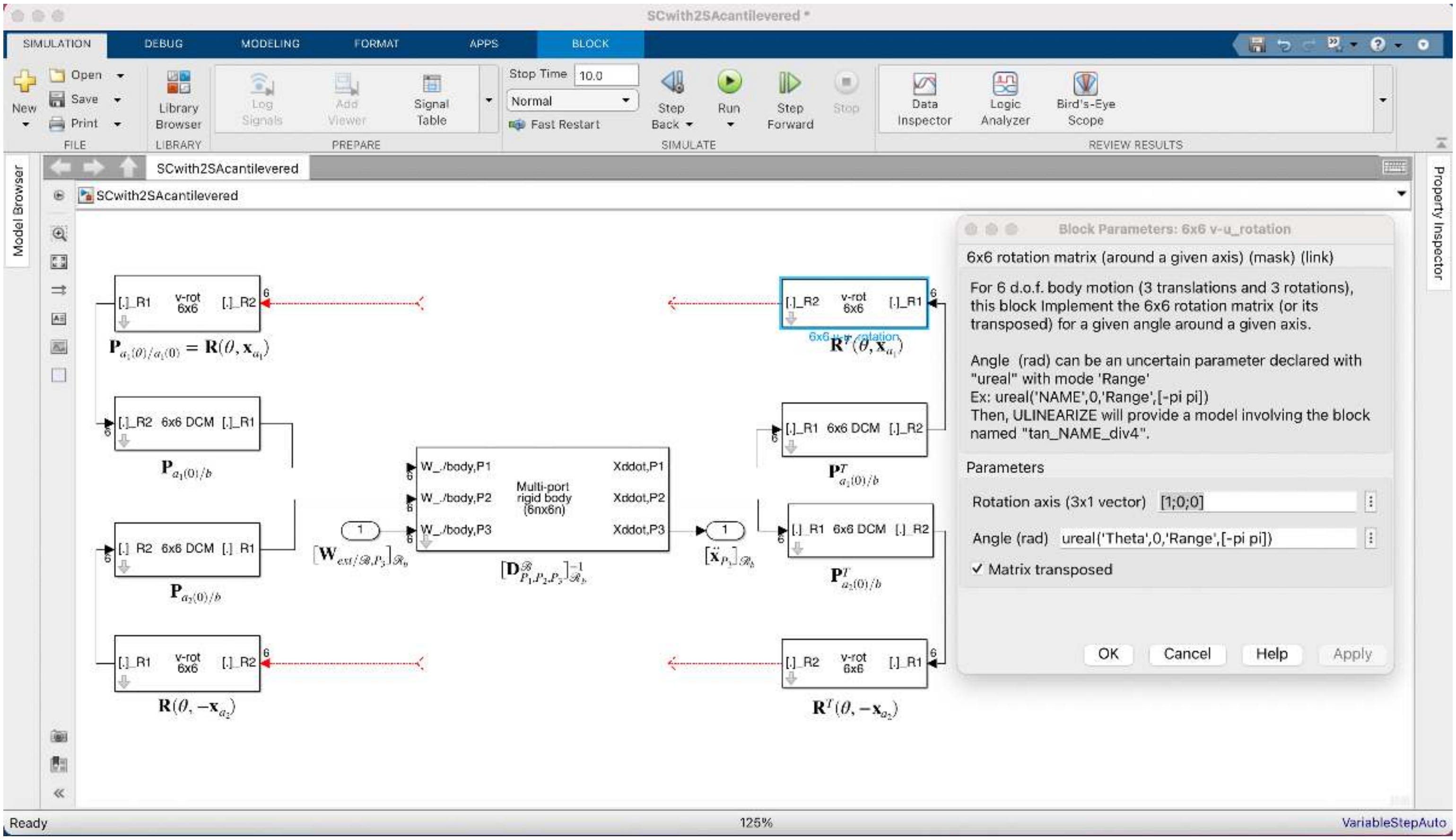
Note that the direct DCMs act on the inputs of the model $[\mathbf{D}^{\mathscr{B}}_{P_1,P_2,P_3}]^{-1}_{\mathscr{R}_b}$ while the DCMs transposed are on the outputs of this model.

The DCM between frames $\mathscr{R}_{a_1}(0)$ and $\mathscr{R}_{a_1}(\theta)$ is associated to the rotation of a varying angle θ around the axis $\mathbf{x}_{a_1}$:

$$\mathbf{P}_{a_1(\theta)/a_1(0)} = \mathbf{R}(\theta, \mathbf{x}_{a_1}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} .$$

In the same way $\mathbf{P}_{a_2(\theta)/a_2(0)} = \mathbf{R}(\theta, -x_{a_2})$.

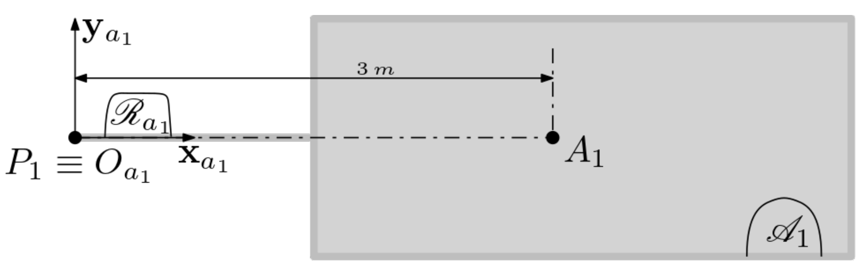
The DCMs can be modeled with the **`6x6 v-u_rotation`** of the sub-library **`3/6 dof DCM`** of the **`SDTlib`**:



Note that the angle θ is declared as an uncertain parameter (**ureal**) varying between $-\pi$ and π. This block uses the parametrization $\sigma_4 = \tan(\theta/4)$ to express the DCM as an LFT (Linear Franctional Transformation) in σ_4. The detail of this parametrization is explained in:

```
web Doc3vurotation.html -new
web Doc3zurotation.html -new
```

Data relative to the solar array $\mathscr{A}_1$:



All the data are expressed in the body frame $\mathscr{R}_{a_1}$:

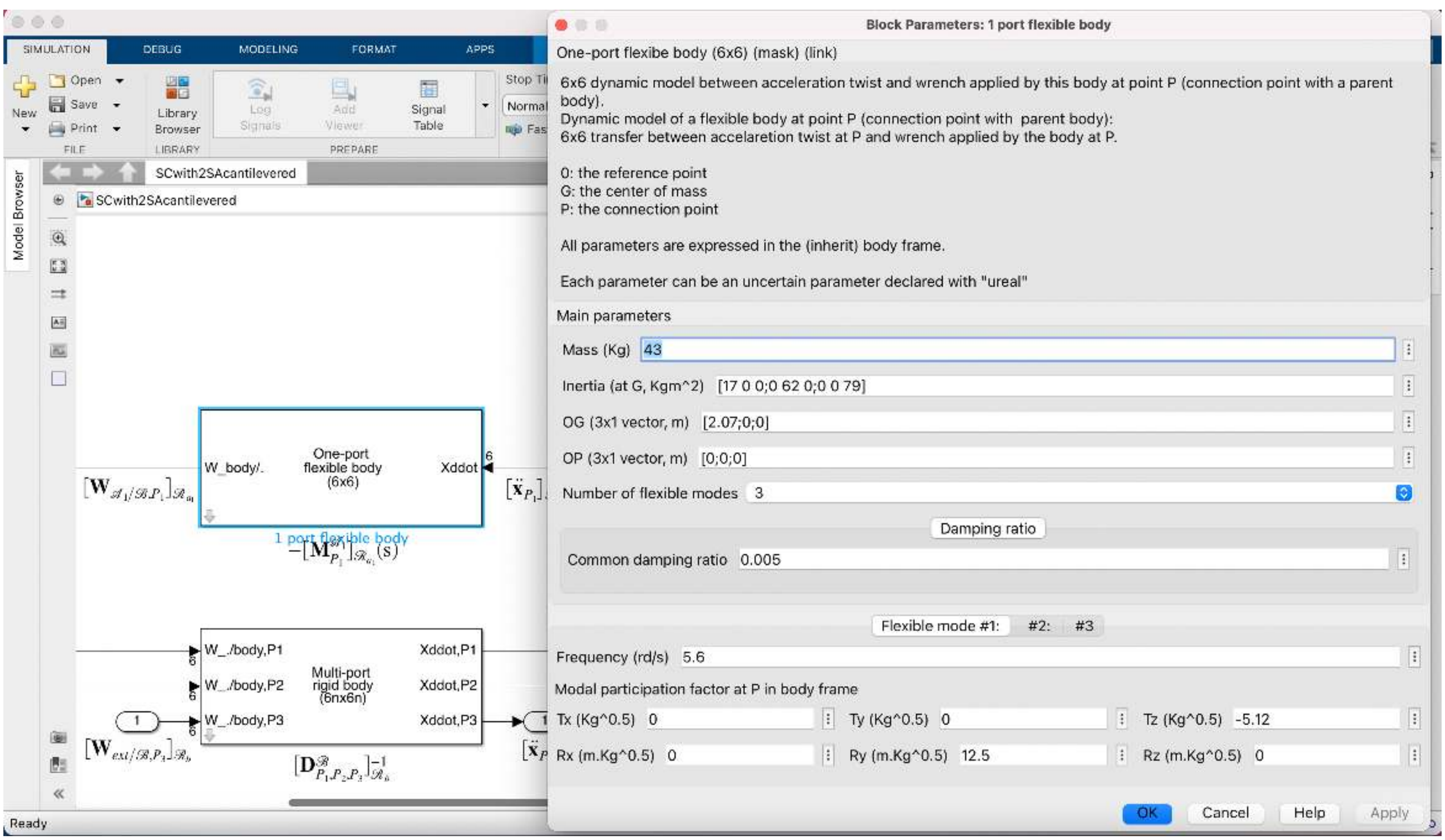
- geometry (in m): $[\overrightarrow{O_{a_1}A_1}]_{\mathscr{R}_{a_i}} = [2.07 \quad 0 \quad 0]^T$, $[\overrightarrow{O_{a_1}P_1}]_{\mathscr{R}_{a_i}} = [0 \quad 0 \quad 0]^T$
- mass: $m^{\mathscr{A}1} = 43\,(Kg)$,

- 3x3 inertia tensor at A_1: $[\mathbf{J}_{A_1}^{\mathscr{A}_1}]_{\mathscr{R}_{a_1}} = \begin{bmatrix} 17 & 0 & 0 \\ 0 & 62 & 0 \\ 0 & 0 & 80 \end{bmatrix}$ $(Kg\,m^2)$,
- the frequencie ω_j and the 1×6 modal participation factor vector $\mathbf{l}_{j,P_1}$ at the connection point P_1 of the j-th solar array flexible mode.
- the common damping ratio for the flexible modes $\xi = 0.005$.

3 flexible modes are considered for $\mathscr{A}_1$:

j	$\omega_j\,(rad/s)$	$[\mathbf{l}_{j,P_1}]_{\mathscr{R}_{a_1}} = [T_x, T_y, T_z, R_x, R_y, R_z]\ (\sqrt{Kg}\,, m\sqrt{Kg})$
1	5.6	$[0 \quad 0 \quad -5.12 \quad 0 \quad 12.5 \quad 0]$
2	19.3	$[0 \quad 0 \quad 0 \quad -3.84 \quad 0 \quad 0]$
3	35.4	$[0 \quad 0 \quad -2.97 \quad 0 \quad 2.51 \quad 0]$

The dialog box of the block **`1 port flexible body`** allows to fill all the data:



As explained in its documentation:

```
web Doconeportflexiblebody.html -new
```

this block:

- computes the dynamic model $[\mathbf{M}_{P_1}^{\mathscr{A}_1}]_{\mathscr{R}_{a_1}}(\mathrm{s})$ of the body $\mathscr{A}_1$ at the point P_1 and projected in the body frame $\mathscr{R}_{a_1}$, that is the matrix transfer from the acceleration twist $[\ddot{\mathbf{x}}_{P_1}]_{\mathscr{R}_{a_1}}$ imposed at point P_1 by the main body $\mathscr{B}$ to the reaction wrench $[\mathbf{W}_{\mathscr{B}/\mathscr{A}_1,P_1}]_{\mathscr{R}_{a_1}}$ applied by body $\mathscr{B}$ on the body $\mathscr{A}_1$,
- and applies the action/reaction principle: $[\mathbf{W}_{\mathscr{A}_1/\mathscr{B},P_1}]_{\mathscr{R}_{a_1}} = -[\mathbf{W}_{\mathscr{B}/\mathscr{A}_1,P_1}]_{\mathscr{R}_{a_1}}$.

Assuming the damping ratios are null ($\xi = 0$), one can express the model as a 6×6 transfer matrix:

$$\mathbf{M}_{P_1}^{\mathscr{A}_1}(\mathrm{s}) = \mathbf{D}_{P_1 0}^{\mathscr{A}_1} + \sum_{j=1}^{n=3} \frac{\mathbf{l}_{j,P_1}^T \mathbf{l}_{j,P_1} \omega_j^2}{\mathrm{s}^2 + \omega_j^2} .$$

(note that this equation is intrinsic and can be projected in any frame.)

$\mathbf{D}_{P_1 0}^{\mathscr{A}_1}$ is the 6×6 residual mass "rigidly" attached to the point P_1.

The DC gain of the model $\mathbf{M}_{P_1}^{\mathscr{A}_1}(0)$ is always equal to the total mass matrix $\mathbf{D}_{P_1}^{\mathscr{A}_1}$ of the body $\mathscr{A}_1$ at the point P_1:

$$\mathbf{M}_{P_1}^{\mathscr{A}_1}(0) = \mathbf{D}_{P_1}^{\mathscr{A}_1} = \boldsymbol{\tau}_{A_1 P_1}^T \underbrace{\begin{bmatrix} m^{\mathscr{A}_1}\mathbf{1}_3 & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \mathbf{J}_{A_1}^{\mathscr{A}_1} \end{bmatrix}}_{\mathbf{D}_{A_1}^{\mathscr{A}_1}} \boldsymbol{\tau}_{A_1 P_1} .$$

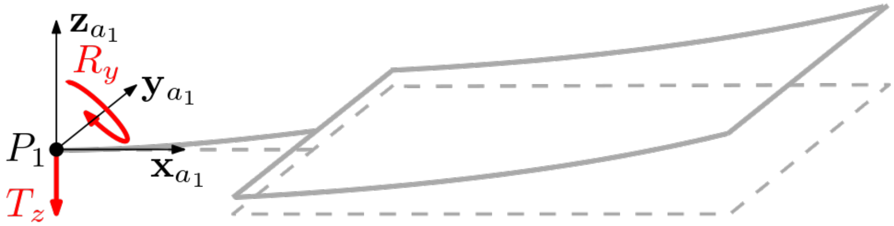
Thus $\mathbf{D}_{P_1 0}^{\mathscr{A}_1} = \mathbf{D}_{P_1}^{\mathscr{A}_1} - \sum_{j=1}^{n=3} \mathbf{l}_{j,P_1}^T \mathbf{l}_{j,P_1}$.

Since it is possible to declare the mass $m^{\mathscr{A}_1}$, the components of the inertia tensor $\mathbf{J}_{A_1}^{\mathscr{A}_1}$, the 3 components of the vectors $\overrightarrow{O_{a_1}A_1}$, $\overrightarrow{O_{a_1}P_1}$ and all the components of the modal participation factors $\mathbf{l}_{j,P_1}$ as uncertain parameters (**`ureal`**), the block **`1 port flexible body`** includes a worst-case analysis to check that the residual mass $\mathbf{D}_{P_1 0}^{\mathscr{A}_1}$ is always a definite positive matrix for any parametric configurations.

A short focus on modal participation factors:

In the block **`1 port flexible body`** the modal participation factors must be provided by the user. This task could seem cumbersome. To better understand the physical meaning of the modal participation factors, one can interpret the data provided for the 3 flexible modes of this numerical application (see table above) on the following figures:

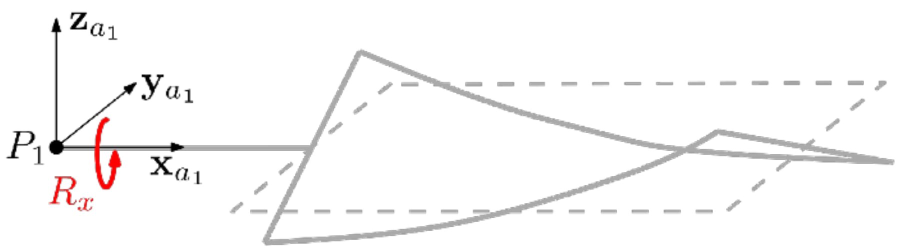
- **mode # 1** ($5.6\, rd/s$):



This mode is a pure bending mode along the $\mathbf{z}$-axis of the solar panel. When excited this mode creates, at the cantilevered point P_1, a negative force T_z on the body $\mathscr{B}$ along $\mathbf{z}_{a_1}$ and a positive torque R_y around $\mathbf{y}_{a_1}$. Reciprocally, an acceleration at the cantilever point P_1 along T_z (translation) or around R_y (rotation) will excite (positively) this mode. The contribution of this mode, in terms of Kg and $Kg\,m^2$, to the total mass matrix $\mathbf{D}_{P_1}^{\mathscr{A}_1}$ is:

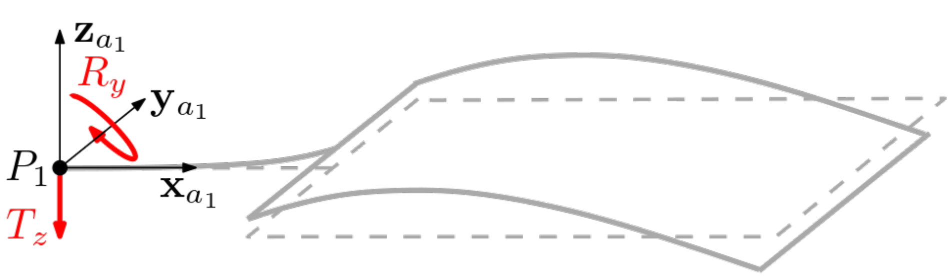
$$\mathbf{l}_{1,P_1}^T \mathbf{l}_{1,P_1} = \begin{bmatrix} 0 \\ 0 \\ -5.12 \\ 0 \\ 12.5 \\ 0 \end{bmatrix} [0 \;\; 0 \;\; -5.12 \;\; 0 \;\; 12.5 \;\; 0] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 26.2144\,(Kg) & 0 & -64\,(Kg\,m) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -64\,(Kg\,m) & 0 & 156.25\,(Kg\,m^2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

- **mode # 2** $(19.3.6\,rd/s)$:



This mode is pure torsion mode around the $\mathbf{x}$-axis of the solar array. It creates a torque R_x on the main body around the $\mathbf{x}_{a_1}$-axis and can be excited by an angular acceleration around this axis. Its contribution to the total mass matrix is $3.84^2\,(Kg\,m^2)$ around the $\mathbf{x}_{a_1}$-axis.

- **mode # 3** $(35.4\,rd/s)$:



This mode is the second bending mode along the $\mathbf{z}$-axis of the solar panel. This mode acts as the mode #1 .

The contribution of this mode, in terms of Kg and $Kg\,m^2$, to the total mass matrix $\mathbf{D}_{P_1}^{\mathcal{A}_1}$ is:

$$\mathbf{l}_{3,P_1}^T \mathbf{l}_{3,P_1} = \begin{bmatrix} 0 \\ 0 \\ -2.97 \\ 0 \\ 2.51 \\ 0 \end{bmatrix} [0 \;\; 0 \;\; -2.97 \;\; 0 \;\; 2.51 \;\; 0] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8.8209\,(Kg) & 0 & -7.4547\,(Kg\,m) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -7.4547\,(Kg\,m) & 0 & 6.3001\,(Kg\,m^2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Note that the full version of the SDTlib includes several blocks where the modal participation factors are self-provided (not provided by the user):

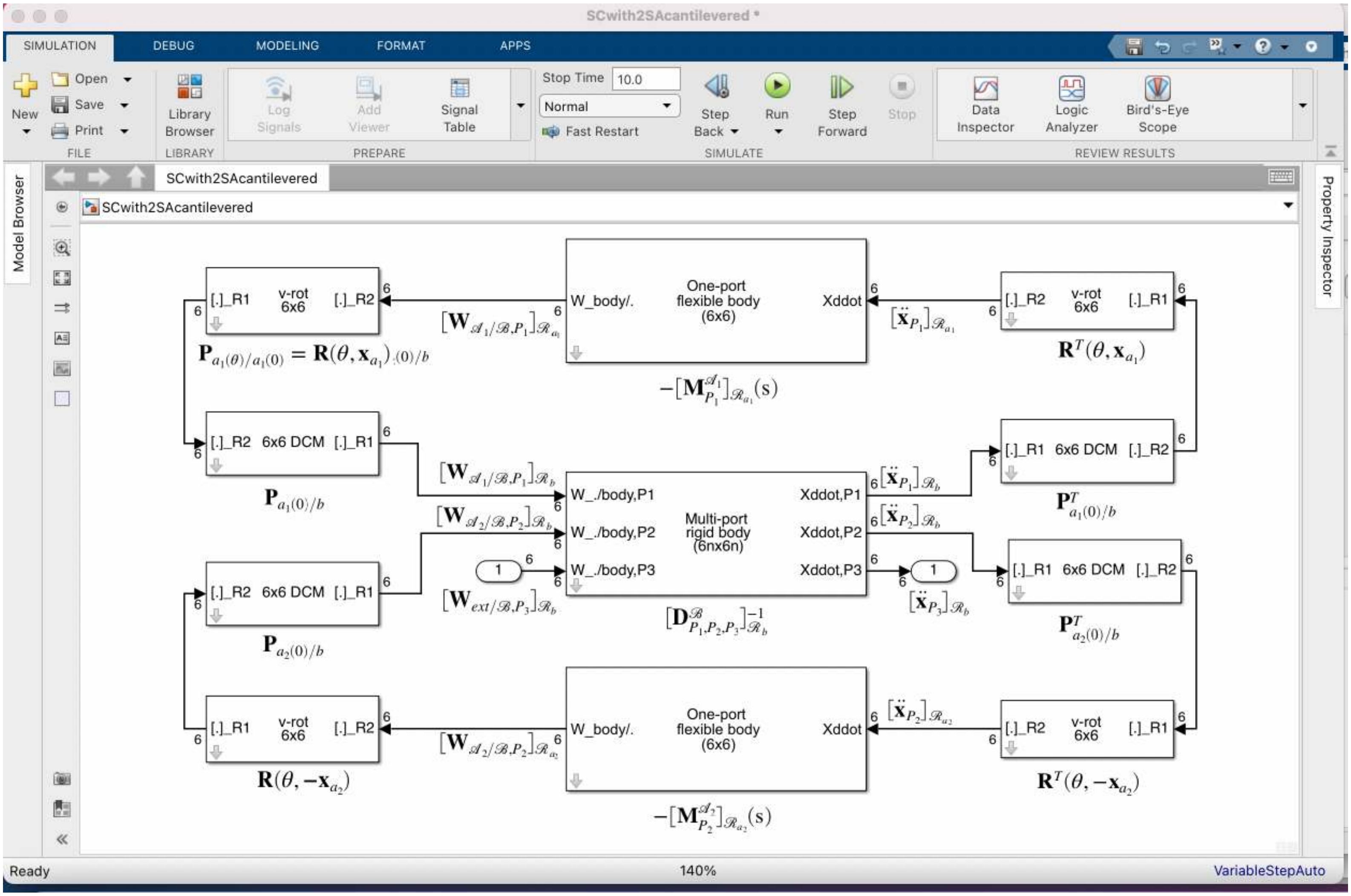
- the blocks **`1 port Nastran body`** and **`N ports Nastran body`** include a Nastran/SDTlib interface to read the modal participation factor in the **`*.f06`** file for any bodies with complex shapes,
- the blocks **2 port flexible beam** and **mult port flexible plate** include an anlytical models of beam and plates where the modal participation factors are analytically computed inside the block,
- the block **`multi-port FEM Kirchoff plate`** includes a finite-element model of a plate and computes the associated modal participation factors.

Data relative to the solar array $\mathcal{A}_2$:

Due to the symmetry, this block is identical to the previous one.

3. The whole 6×6 dynamic model

The whole model is then described by the following SIMULINK file **`SCwith2SAcantilevered.slx`**:




```
SCwith2SAcantilevered                    % the SIMULINK file of the model
Gu=ulinearize('SCwith2SAcantilevered')
```

```
Gu =

  Uncertain continuous-time state-space model with 6 outputs, 6 inputs, 12 states.
  The model uncertainty consists of the following blocks:
    tan_Theta_div4: Uncertain real, nominal = 0, range = [-1,1], 32 occurrences

Type "Gu.NominalValue" to see the nominal value, "get(Gu)" to see all properties, and "Gu.Uncertainty" to
```

One can check that the model have 6 inputs: the external wrench $[\mathbf{W}_{ext/\mathscr{B},B}]_{\mathscr{R}_b}$ (6 components) and 6 outputs:

the dual vector of acceleration of point B $[\ddot{\mathbf{x}}_B]_{\mathscr{R}_b}$ (6 components),

and 12 states or 6 modes: 3 flexible modes per appendage $\times$ 2.

This model depends on the uncertain parameter $\sigma_4 = \tan\dfrac{\theta}{4}$ (**`tan_Theta_div4`**) with 32 occurences.

4. First analyses

It is now possible to:

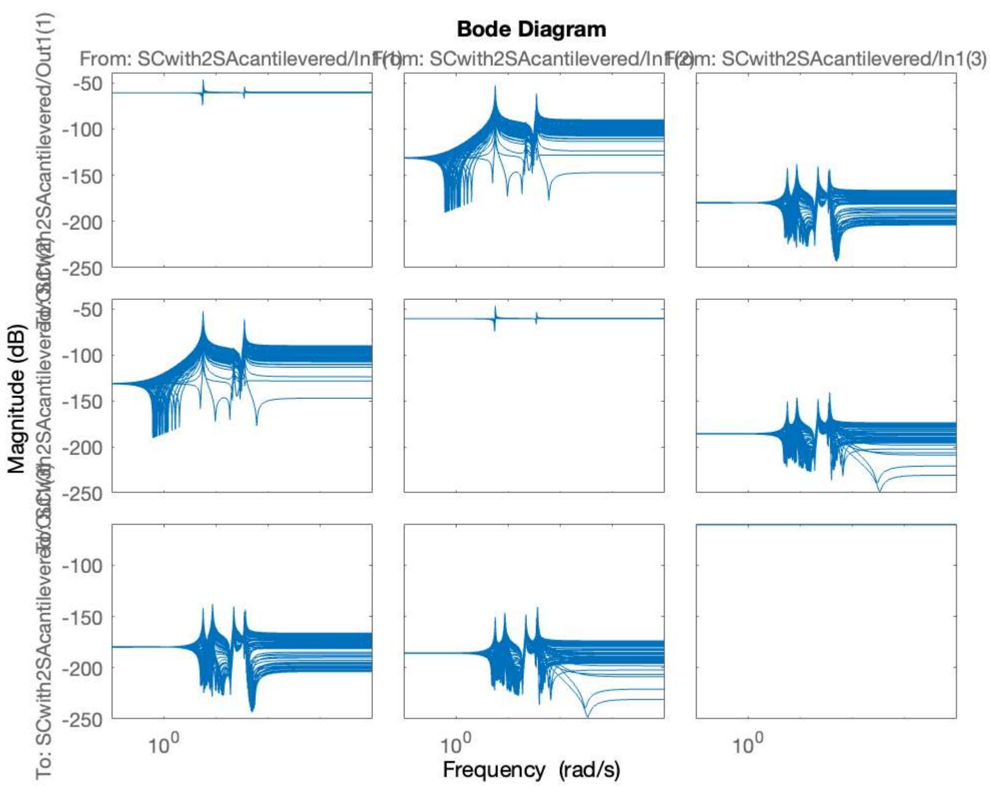
- compute the total 6×6 mass/inertia of the spacecraft in the nominal configuration: $[\mathbf{D}_B^{\mathcal{S}\mathcal{C}}]_{\mathcal{R}_b}(0,0)$ (note the function **dcgain** does not consider parametric variations):

```
inv(dcgain(Gu))
```

```
ans = 6×6
10^3  ×
    1.0860         0   -0.0000    0.0000   -0.0000    0.0086
   -0.0000    1.0860   -0.0000    0.0000    0.0000    0.0043
   -0.0000   -0.0000    1.0860   -0.0086   -0.0043   -0.0000
   -0.0000    0.0000   -0.0086    0.7679    0.0014    0.0020
    0.0000    0.0000   -0.0043    0.0014    0.7662   -0.0010
    0.0086    0.0043   -0.0000    0.0020   -0.0010    0.1151
```

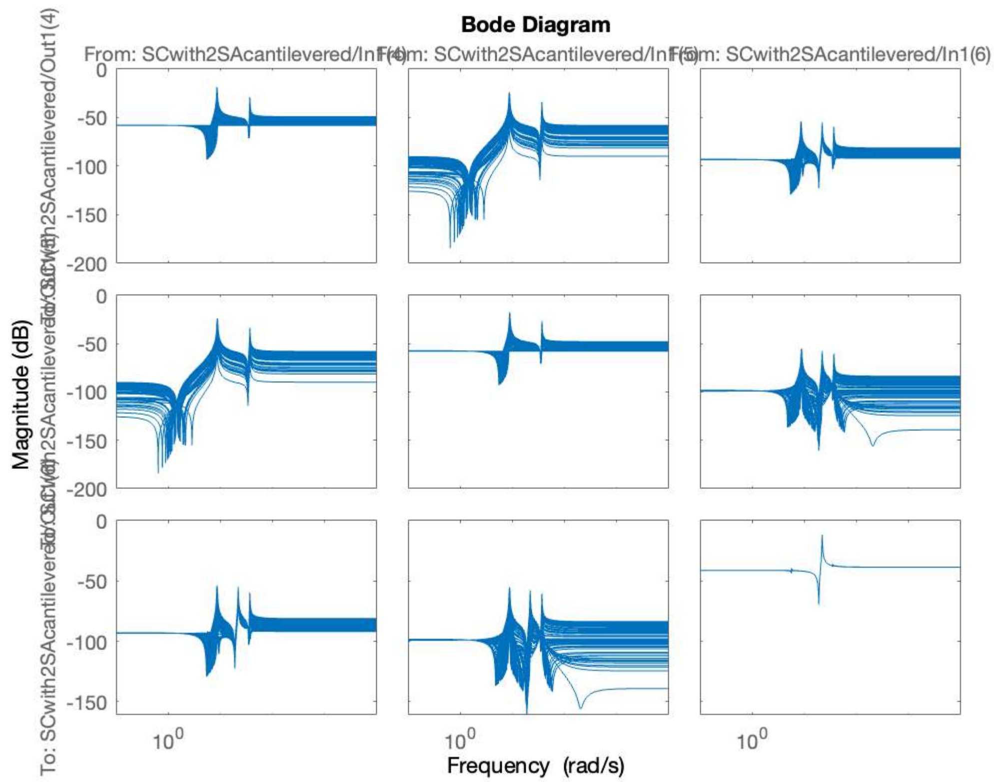
- plot the frequency-domain response (*Bode* magnitude) of the 3×3 transfer between the force applied to the spacecraft and its translation acceleration at B, for 100 random samples in the parametric space:

```
rng(1) % to freeze the random number sequence from one run to the next.
figure
bodemag(usample(Gu(1:3,1:3),100))
```



- plot the frequency-domain response (Bode magnitude) of the 3×3 transfer between the torque applied to the spacecraft at B and its angular acceleration, for 100 random samples in the parametric space:

```
figure
bodemag(usample(Gu(4:6,4:6),100))
```



5. Remarks

- In the various blocks of the **6 dof bodies** sub-library, the dialog box requires to fill for a given body $\mathscr{C}$: (i) the mass $m^{\mathscr{C}}$, (ii) the 3×3 inertia tensor $[\mathbf{J}_G^{\mathscr{C}}]_{\mathscr{R}_c}$ of the body $\mathscr{C}$ at its center of mass G and (iii) the vector $[\overrightarrow{OG}]_{\mathscr{R}_c}$ from a reference point O to the center of mass G. In some application, the only data available is the 6×6 total mass/inertia matrix $[\mathbf{D}_O^{\mathscr{C}}]_{\mathscr{R}_c}$ (also called direct dynamic model) at a reference point O. Then, one can apply the dynamic model transport property: $\mathbf{D}_O^{\mathscr{C}} = \boldsymbol{\tau}_{GO}^T \mathbf{D}_G^{\mathscr{C}} \boldsymbol{\tau}_{GO} = \begin{bmatrix} m^{\mathscr{C}}\mathbf{1}_3 & m^{\mathscr{C}}(^*\overrightarrow{GO}) \\ -m^{\mathscr{C}}(^*\overrightarrow{GO}) & \underbrace{\mathbf{J}_G^{\mathscr{C}} - m^{\mathscr{C}}(^*\overrightarrow{GO})^2}_{\mathbf{J}_O^{\mathscr{C}}} \end{bmatrix}$ which is valid in any frame to recover the required data: $m^{\mathscr{C}}$, $\overrightarrow{OG}$, $\mathbf{J}_G^{\mathscr{C}} = \mathbf{J}_O^{\mathscr{C}} + m^{\mathscr{C}}(^*\overrightarrow{OG})$ where $(^*\overrightarrow{OG})$ is the antisymmetric matrix associated to the vector $\overrightarrow{OG}$. That can be easily implemented thanks to the function **`antisym.m`** as illustrated in the following example.

```
D_CatO=[50 0 0 0 50 0;0 50 0 -50 0 100;0 0 50 0 -100 0;...
```

```
          0 -50 0 67 0 -100;50 0 -100 0 312 0;0 100 0 -100 0 280]
```

```
D_CatO = 6×6
    50     0     0     0    50     0
     0    50     0   -50     0   100
     0     0    50     0  -100     0
     0   -50     0    67     0  -100
    50     0  -100     0   312     0
     0   100     0  -100     0   280
```

```
m_C=D_CatO(1,1)
```

```
m_C = 50
```

```
OG=[D_CatO(2,6);-D_CatO(1,6);D_CatO(1,5)]/m_C
```

```
OG = 3×1
     2
     0
     1
```

```
J_CatG=D_CatO(4:6,4:6)+m_C*antisym(OG)^2
```

```
J_CatG = 3×3
    17     0     0
     0    62     0
     0     0    80
```